Phys220

NAME ______________________ ID Number ______________

1- Consider two coaxial cylindrical conducting surfaces. Let a be the radius of the inner surface, b the radius of the outer surface, and l the length of the system. Now, we fill the space between the two surfaces with silicon. Determine the total resistance of silicon between the two conducting surfaces. Note that the resistivity of silicon is 640 ohms.

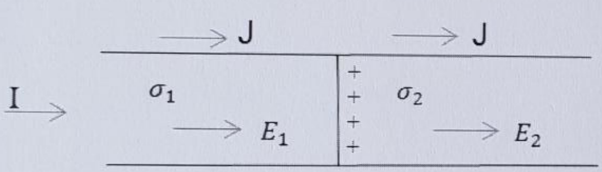
2- Consider two parallel long straight wires separated by a distance d carrying currents i1 and i2. Let i1=12 sin wt and i2=18sin(wt-2π/3).

a- What is the maximum force per meter that these two currents exert on each other?

b- Determine whether this maximum force is attractive or repulsive.

3- Determine the force f exerted by a current i1 in a long straight wire on a segment MN carrying a current i2 perpendicular to i1. Assume that point O is the intersection between the straight prolongation of segment MN and the long straight wire, and OM=m and ON=n.

4- A bar is made of two materials of different conductivities σ_1 and σ_2. A steady current I is flowing through the bar which also has a constant current density J. This implies that there is a static charge at the junction which has area A.



a- Explain using Gauss's law the fact that there is static charge at the junction.

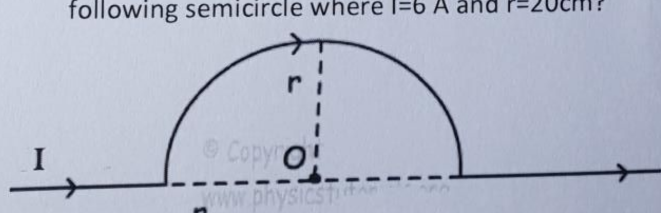
b- Find the total amount of charge at the junction in terms of the current and the conductivities.

5- On a nylon filament 0.01cm in diameter and 4 cm long there are 5×10^8 extra electrons distributed uniformly over the surface. What is the electric field strength at the surface of the filament:

a- In the rest frame of the filament?

b- In a frame where the filament is moving at a speed 0.9c in a direction parallel to its length?

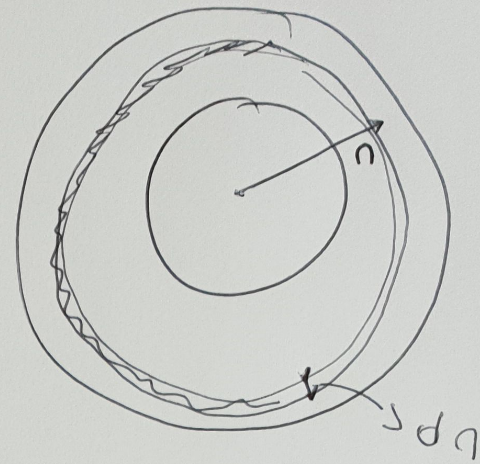
6- Use Bio-Savart law to find the magnetic field strength at the center of a circular coil of radius r carrying a current I. What is the magnitude and direction of the magnetic field at the center of the following semicircle where I=6 A and r=20cm?



1)

this ~~the~~ is the resistivity of Si

$\Rightarrow dR = \dfrac{\rho \, dr}{s}$

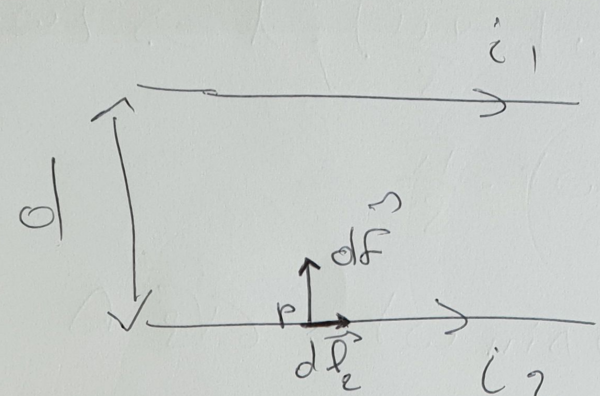


where s is the surface of one element that the current traverses.

$$\Rightarrow dR = \frac{\rho \cdot dr}{2\pi r \cdot l} \Rightarrow R = \frac{\rho}{2\pi l} \int_a^b \frac{dr}{r}$$

$$\Rightarrow R = \frac{\rho}{2\pi l} \log \frac{b}{a}$$

2)



$\vec{df} = i_2 \cdot \vec{dl_2} \times \vec{B_1}$

$B_1 = \dfrac{\mu_0 i_1}{2\pi d}$

$\Rightarrow$ ~~$\vec{df} = i_2 \, dl_2 \, B_1$~~

~~df =~~ the two ~~cu~~ currents are in the same direction $\Rightarrow \vec{df}$ is attractive

$df = i_2 \, dl_2 \, B_1 = \dfrac{\mu_0 i_1 i_2}{2\pi d} \cdot dl_2$

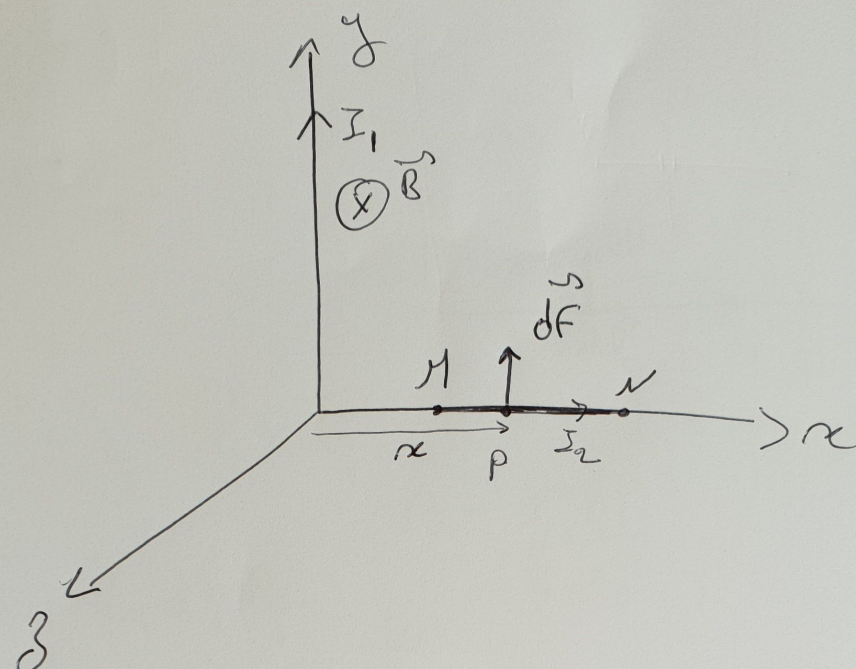
$\Rightarrow f = \dfrac{\mu_0 i_1 i_2}{2\pi d}$

$\Rightarrow f = \dfrac{\mu_0}{2\pi d} \cdot 12 \sin \omega t \cdot 18 \sin\left(\omega t - \dfrac{2\pi}{3}\right)$

but $\sin a \cdot \sin b = \dfrac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$

$\Rightarrow f \propto \cos\left(2\omega t - \dfrac{2\pi}{3}\right) \Rightarrow$ maximum if $\cos\left(2\omega t - \dfrac{2\pi}{3}\right) = 1$

3)



The force exerted on the element dx at distance x:

$$d\vec{F} = I_2 \, dx \, \hat{i} \wedge \vec{B} \quad (\vec{i} \text{ is a unit vector along } x)$$

$$\vec{B} = \frac{\mu_0 I_1}{2\pi x}(-\vec{k}) \quad (\vec{k} \text{ is a unit vector along } z)$$

$$\Rightarrow d\vec{F} = I_2 \, dx \, \vec{i} \wedge \left(-\frac{\mu_0 I_1}{2\pi x} \cdot \vec{k}\right)$$

$$d\vec{F} = \frac{\mu_0 I_1 I_2}{2\pi} \frac{dx}{x} \vec{j}$$

$$\Rightarrow \vec{F} = \frac{\mu_0 I_1 I_2}{2\pi} \int_m^n \frac{dx}{x} \vec{j} \Rightarrow \boxed{\vec{F} = \frac{\mu_0 I_1 I_2}{2\pi} \ln \frac{n}{m} \cdot \vec{j}}$$

4) Gauss's law tells us that $\int E \cdot da = \frac{q}{\varepsilon_0}$

at the junction using a pillbox with area A

for the junction

$$A(E_2 - E_1) = \frac{Q}{\varepsilon_0}$$

since $E_2 \neq E_1$, $Q \neq 0$

b) $E_1 = \frac{J}{\sigma_1}$ $E_2 = \frac{J}{\sigma_2}$

$$A\left(\frac{J}{\sigma_2} - \frac{J}{\sigma_1}\right) = \frac{Q}{\varepsilon_0} \Rightarrow Q = \varepsilon_0 AJ\left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1}\right)$$

$$= \varepsilon_0 I\left(\frac{1}{\sigma_1} - \frac{1}{\sigma_1}\right)$$

5) excess charge = $Q = Ne = 5\times10^{8}\times1.6\times10^{-19}$ C

$= 8\times10^{-11}$ C

$\Rightarrow E = \frac{\lambda}{4\pi\varepsilon_0 r} = \frac{Q/\ell}{4\pi\varepsilon_0 r} = 7.2\times10^{5}$ V/m radially towards the wire

b) in this frame $E' = \gamma E = \frac{1}{\sqrt{1-0.9^2}} E$

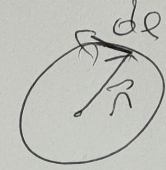
$= 2.29\times E$

$= 1.65\times10^{6}$ V/m

radially toward the wire

6) a)

$$B = \frac{\mu_0}{4\pi}\int \frac{I \times \hat{r}}{r^2}\, dl = \frac{\mu_0}{4\pi}\int \frac{I\, dl}{r^2} \times \hat{r}$$



$$= \frac{\mu_0 I}{4\pi r^2}\int dl$$

since I is constant, r is constant and $|dl \times \hat{r}| = dl$

$$= \frac{\mu_0 I}{4\pi r^2}\cdot 2\pi r = \frac{\mu_0 I}{2r}$$

b) only contribution is from semicircle

$$B = \frac{1}{2}\left(\frac{\mu_0 I}{2r}\right) = \frac{1}{2}\cdot\frac{4\pi\times 10^{-7}\times 6}{2\times 0.2} = \frac{6\pi}{0.2}\times 10^{-7}$$

$$\simeq 9.42\times 10^{-7}$$

B directed in page.